

IMPROVING HABITS OF MIND TO PROMOTE ADVANCED MATHEMATICAL THINKING

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**Presented at ICREM 5
ITB - Bandung
October 23, 2011**

INTRODUCTION

- Research found that students have difficulties on mathematical proving (Moore, 1994).
- Moore method is a learning method that encourage students to think and produce mathematical ideas independently, and present it orally and written (Mahavier, May, Parker, 2006) .
- An experiment using modified Moore method reported that a supportive environment did not fulfill for attaining students' mathematical understanding and proving abilities yet (Maya, 2011).
- Students still obtained a low grade on mathematical understanding and proving abilities even though they performed positive opinion on the mathematics learning environment (Maya, 2011)

INTRODUCTION (cont.)

- 19,2% students in MMT class could master 70% in constructing proof. Only 13,3% students in Conventional class could master 70% in constructing proof (Maya, 2011)
- 30% students could master 75% in constructing proof and 3% of it achieved ideal score. (Hanna & Jahnke, 1996)
- Every course or academic experience should be used as an opportunity to help students develop good general habits of mind (Cuoco, Goldenberg & Mark, 1996)
- Mathematical thinking is a dynamic process which, by enabling us to increase the complexity of ideas we can handle, expands our understanding (Mason, Burton & Stacey, 2010)
- Ability to prove is one of abilities of advanced mathematical thinking.

Habits of Mind

- Term of habits of mind constitute a strong disposition toward doing intelligently when students face a mathematical problem which its solution could not encountered immediately yet (Costa & Kallick, 2001).
- Students' mathematical habits of mind are predicted to help students to think about mathematics as "the way mathematicians do".
- Some attributes of habits of mind among other things are persisting, questioning and posing problems, applying past knowledge to new situations, gathering data through all senses, thinking and communicating with clarity and precision.

Habits of Mind (cont.)

Costa & Kallick (2001) described 16 habits of mind as:

1. **Persisting:** if a student has persistence it means that he/she continue to do something with determination, even though it is difficult or other people are against it. They don't give up easily.
2. **Questioning and posing problems.**
3. **Applying past knowledge to new situations:** intelligent students learn from the experience. They recall their knowledge and experience to apply it to the new situations.
4. **Gathering data through all senses.**
5. **Thinking and communicating with clarity and precision.**
6. **Listening to Others, with understanding and empathy**

Habits of Mind (cont.)

7. Thinking interdependently
8. Managing Impulsivity
9. Thinking Flexibly
10. Thinking about our thinking (metacognition)
11. Striving about accuracy and precision
12. Creating, Imagining and Innovating
13. Responding with wonderment and awe
14. Taking responsibility risks
15. Finding humor
16. Learning continuously

Moore Method

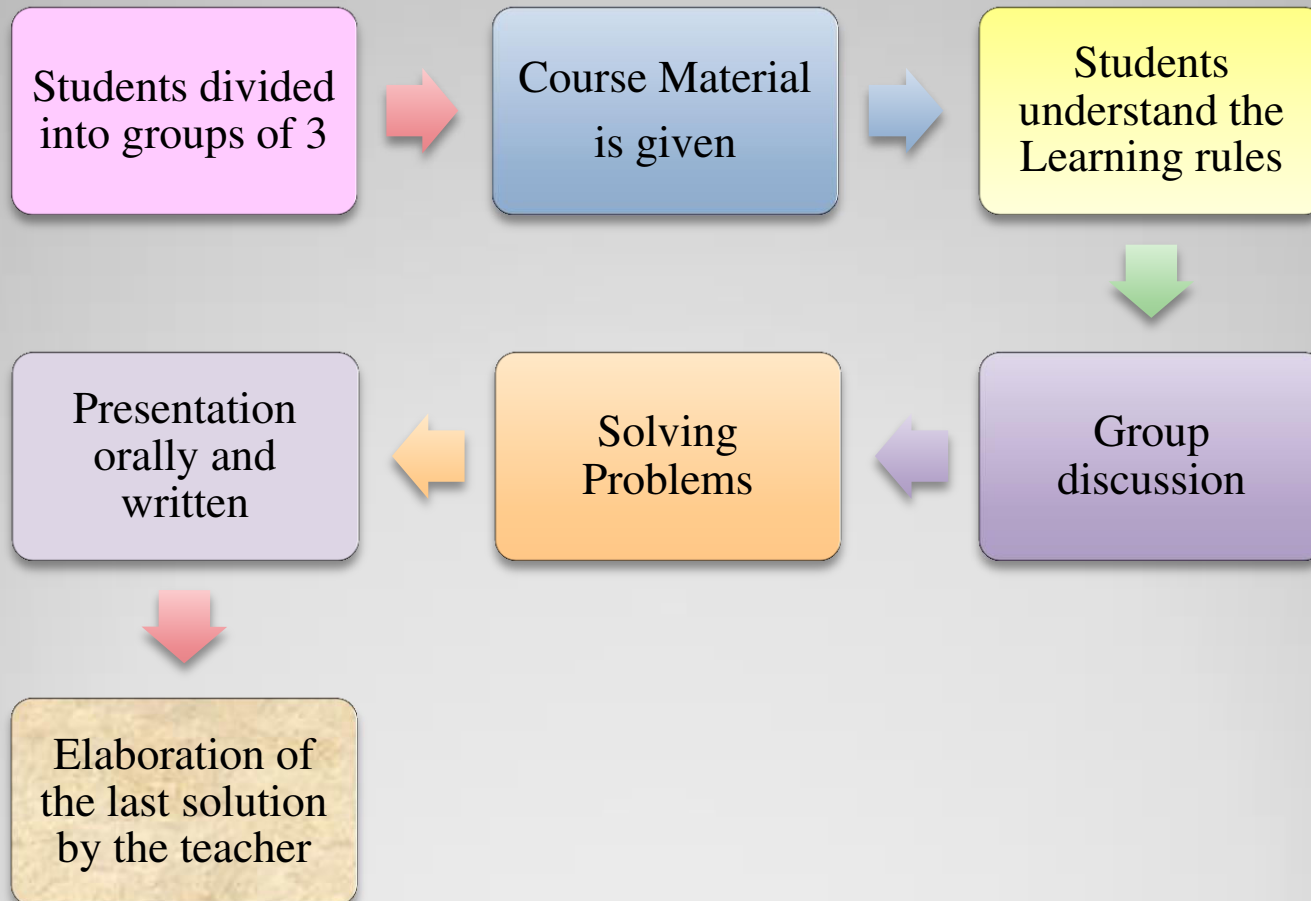
Learning with this method required:

- Course material prepared by the teacher based on textbooks (students prohibited using textbooks);
- Learning in a small class.
(*Modified*: if not possible, a big class can be divided to groups of three);
- Cooperation among individuals are not allowed.
(*Modified*: if in groups, cooperation only inside the groups)
- The teacher/Instructor role as a *guide, a motivator, a fasilitator, or a cheerleader*.

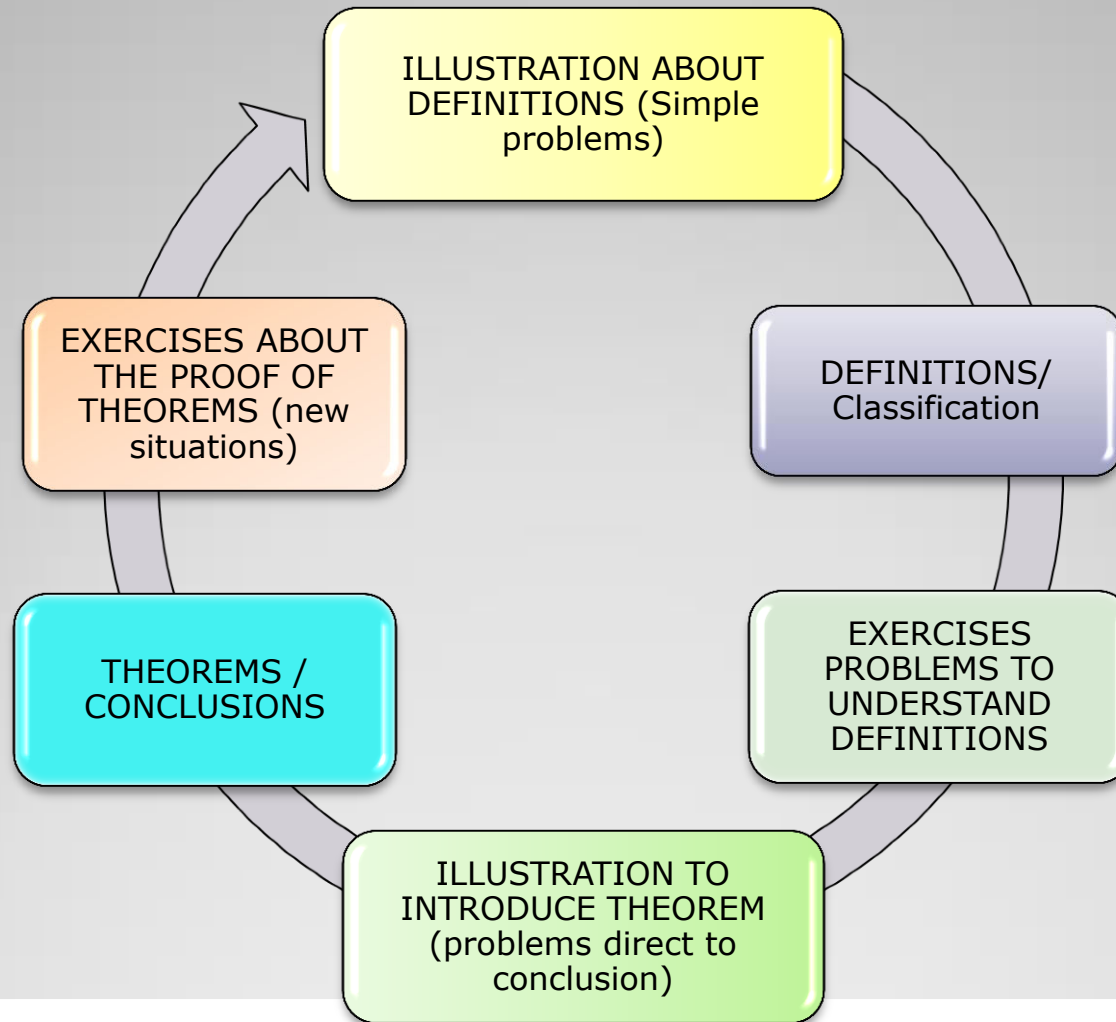
Learning Method Comparison between MMT and Conv.

No.	Characteristic	Learning Method	
		Modified Moore Method	Conventional
1	Course Material	Prepared by the instructor	Using textbooks
2	Learning	Begin with simple problems, construct definitions, exercise related to understanding concepts, illustration to introduce to a new theorems, conclusion, exercise related to proof theorem.	Definition and examples are given, then exercise problems
3	Students Group	Small groups (3 students /groups)	Classical
4	Presentation	Written and orally (always)	Written and orally (sometime)

Sessions in Learning Using MMT



Course Material



Advantages of Learning using MMT

1. Learning using MMT made pleasant learning environment. Students did not afraid to ask to the teacher.
2. Learning using MMT encouraged students to actively involved in learning and motivate them to express their mathematical ideas independently.
3. Cooperation inside and among groups, discussion, presentation orally and written in learning using MMT encouraged students to respect other opinions and able to develop communications ability.

Promoting Advanced Mathematical Thinking

- Based on the finding of a study by using modified Moore method, it is predicted that habits of mind are able to be improved during the lesson.
- During the presentation task phase with their peers in a small group, students will improve their habits of thinking and communicating clearly and precisely.
- In the classroom discussion phase, students are encouraged to pose question or problems to their peers.
- In solving mathematics problems phase, by using students' prior knowledge, they will carry it out in a new situations or a new mathematical concepts so that they obtain meaningful learning.

Promoting Advanced Mathematical Thinking (cont.)

- Students' mathematical habits of mind which improved during the modified Moore teaching approach will promote students' advanced mathematical thinking.
- For motivating students' habits of mind in doing advanced mathematical thinking, it is necessary to present mathematical exercises tasks in such ways so that students will be challenged to reason mathematically, to make sense of mathematics, to encourage conjecturing, to promote problem solving, and to make connections among the ideas and their applications

Examples of Material Course

Theorem 1:

Let ϕ be a ring homomorphism from a ring R to a ring S . Let A be a subring of R and let B be an ideal of S . If A is an ideal of R and ϕ is onto S , then $\phi(A)$ is an ideal.

Problem 1:

Consider the mapping: $\phi : Z_6 \rightarrow Z_6$, which $\phi(x) = 3x$ for every x in Z_6 .

Suppose $A = \{0, 2, 4\}$.

- Do you think A ideal of Z_6 ? Explain your answer.
- Can you investigate whether ϕ onto Z_6 ?
- Can you list all elements of $\phi(A)$?
- Is $\phi(A)$ ideal of Z_6 ? Explain your answer.
- What conclusion will you make regarding your answer? (relate it to the Theorem 1)

Examples of Material Course (cont.)

Theorem 2:

Let ϕ be a ring homomorphism from a ring R to a ring S . ϕ is an isomorphism if and only if ϕ is onto and $\text{Ker}\phi = \{r \in R \mid \phi(r) = 0\} = \{0\}$.

Problem 2:

How would you prove that theorem?

Problem 3:

Consider the mapping $\phi : Z_4 \rightarrow Z_4$, which $\phi(x) = 3x$ for every x in Z_4 .

- Is ϕ an isomorphism? Can you guess and check?
- Can you investigate whether ϕ is onto Z_4 ?
- Let $A = \{x \in Z_4 \mid \phi(x) = 0\} = \text{Ker}\phi$. Can you list all elements of A ?
- From your answers, what do you think about Theorem 2?

Examples of Questions for Promoting AMT (PBS Teacherline, 2006)

1. Questions for helping students to reason mathematically:
 - a. How would you prove that theorem?
 - b. Can you think of a counterexample?
 - c. Is that true for all cases? Explain your answer.
2. Questions for encouraging conjecturing:
 - a. What would happen if ...? What if not?
 - b. Do you see a pattern? Can you explain the pattern?
 - c. What are some possibilities here?
 - d. Can you predict the next one?

Examples of Questions for Promoting AMT (cont.)

3. Questions for promoting problem solving:
 - a. What information do you have?
 - b. What do you need to find out?
 - c. What do you think the answer will be?
4. Questions for making connections among ideas:
 - a. How you relate it to others?
 - b. Can you give me an example of ...?
 - c. What ideas that we have learned before were useful in solving this problem?

Conclusion

- Promoting advanced mathematical thinking need the right material course, the right learning method in supportive environment and habits of mind that support the aim of mathematical learning.
- There are no specific material course, learning method that suitable for all classes. Choosing the material course and learning method based on students conditions and teacher personality (include the high patience) and improving mathematical habits of mind are the best way to encourage students' advanced mathematical thinking.

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THANK YOU

