PROVING ABILITY OF UNDERGRADUATE STUDENTS' OF MATHEMATICS EDUCATION

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INTRODUCTION

- Ability to prove is one of abilities of advanced mathematical thinking.
- Research found that students have difficulties on mathematical proving (Moore, 1994).
- Students still obtained a low grade on mathematical understanding and proving abilities even though they performed positive opinion on the mathematics learning environment (Maya, 2011)

INTRODUCTION (cont.)

- 30% students could master 75% in constructing proof and 3% of it achieved ideal score (Hanna & Jahnke, 1996)
- Students can understand the proofs that are explained by their instructor in the class. But, when they're ask to do it by themselves, they can't do it. They can't use or explore the information in mathematical statement or theorem that will be proven (Barnard, 2000)
- 19.2% students in MMT (Modified Moore Methode) class could master 70% in constructing proof. Only 13.3% students in Conventional class could master 70% in constructing proof (Maya, 2011)

Discussion

- Maya (2011:77) stated that high ability on proving is only achieved by students that have high ability on mathematical understanding.
- The facts showed that some students sometimes do not pay attention to the concepts in definitions or theorems are given. Students are only read definitions and theorems, without understanding it furthermore.
- The lack of conceptual mathematical understanding can not only be blamed on students. The instructor has also big roles in teaching conceptual understanding to the students. The instructor that can not make students understand the concepts will guide the students to learn concepts from the examples are given.
- Students that only learn concepts from the examples, will face difficulties in doing exercises that are different from the examples.

Discussion (continued)

- Selden & Selden (1995) cited Moore statements that knowing a definition and could conver example and non example, it didn’t mean that the students could master language and logical structure for writing proofs directly. Teaching how to compose a proof was a difficult task.

- Maya (2011) stated that acknowledging students habits in learning proofs, they have not been accustomed to construct proofs by themselves. They’re accustomed to write proof that are given by their instructor on the whiteboard, then they understand it and memorize it.

Research by Fischbein (Alibert & Thomas, 2002) has uncovered another aspect to the understanding of proof by students. He presented a correct proof of a theorem:

n3– n is divisible by 6 for every integer n.

The students were then given various questions about the validity of the theorem.

Whilst

- 81 % checked the proof and claimed it to be correct in every detail,
- 68.5% agreed with the theorem and
- 60% considered the generality of the theorem guaranteed by the proof,
- Only 41% of the students accepted all three of these.
- Further only 24.5% accepted the correctness of the proof and at the same time answered that additional checks are not necessary, and
- only 14.5% were completely consistent in their answers.

Proving Difficulties

Moore (1994) and Maya (2011) stated that students have proving difficulties as follow:

- They do not know how to begin a proof
- Their conceptual understanding are not enough to begin proving.
- Lack of understanding of terms or notations in mathematics result they used it improperly.
- They aren’t able to look for connections between one concepts to other concepts.

Proving Difficulties (continued)

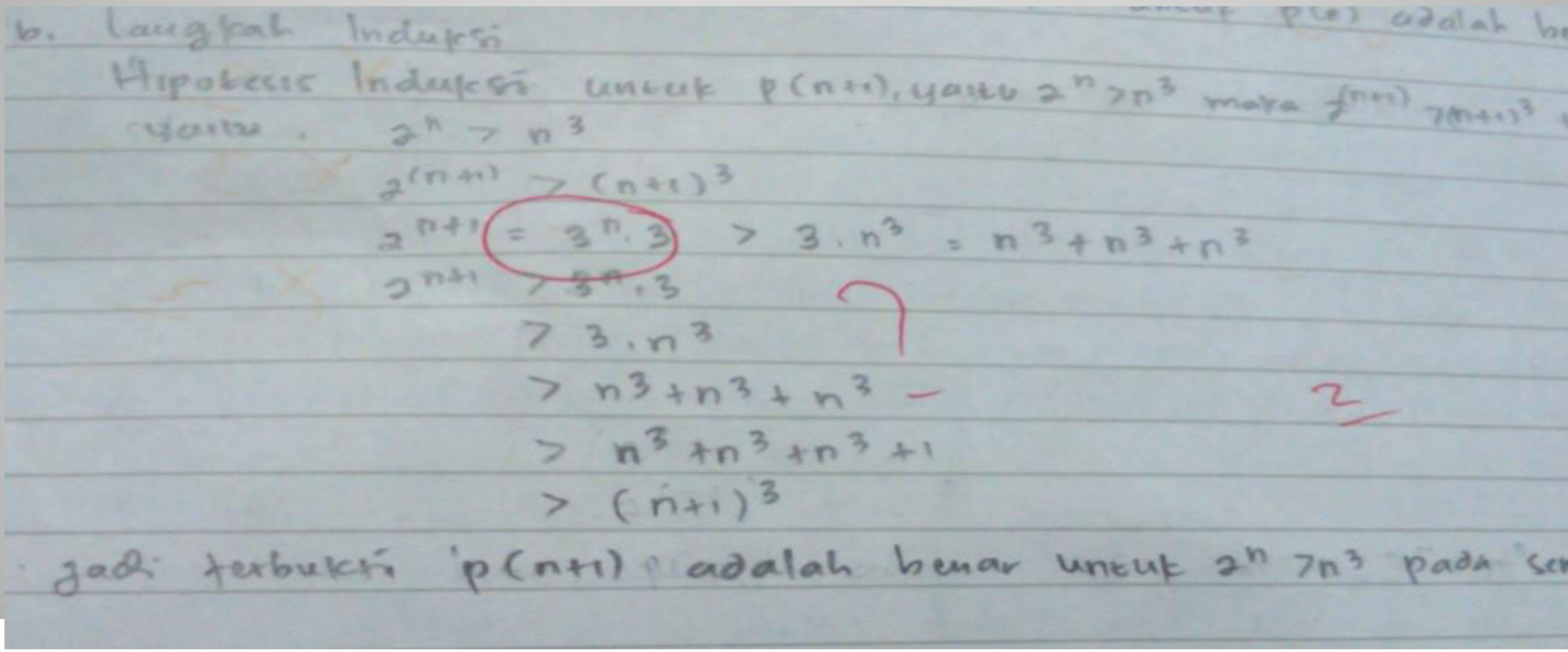
In addition, Selden & Selden (2003:10-15) also stated that there are other mistakes in prove:

- Overextended symbols
- Weakening the theorem
- Notational inflexibility
- Circularity
- The locally unintelligible proof
- Substitution with abandon
- Ignoring and extending quantifiers
- Holes
- Using information out of context

Examples

I take examples from students works in induction step, because in this step students faced difficulties in proving.

Consider these examples: Prove that $2^n > n^3$, for $n \geq 10$.



b. Langkah Induksi

Hipotesis Induksi untuk p(n+1), yaitu $2^n > n^3$ maka $2^{(n+1)} > (n+1)^3$

yaitu.

$2^n > n^3$

$2^{(n+1)} > (n+1)^3$

$2^{n+1} = 3^n \cdot 3 > 3 \cdot n^3 = n^3 + n^3 + n^3$

$2^{n+1} > 3^n \cdot 3$

$> 3 \cdot n^3$

$> n^3 + n^3 + n^3$

$> n^3 + n^3 + n^3 + 1$

$> (n+1)^3$

jadi terbukti p(n+1) adalah benar untuk $2^n > n^3$ pada sem

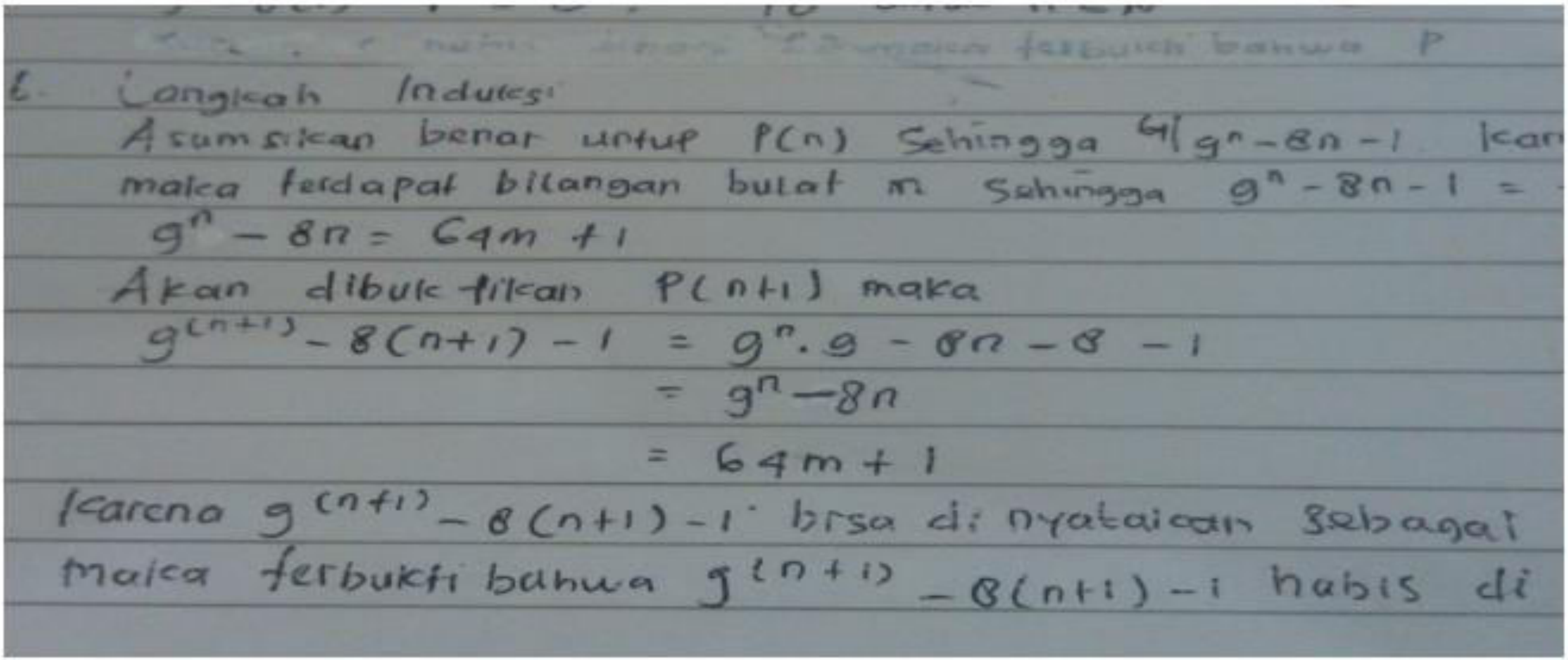
From that student's proof, we can see that students have difficulty on proving. They want to finish proving soon, without paying attention on rational argumentation. In terms of Selden & Selden (2003), students want to jump to conclusion soon.

Below is alternative proof construction for that example.

$$
\begin{aligned}
2^{n+1} &= 2.2^n \\
2.2^n &> 2.n^3 && \text{(based on induction basis)} \\
&= n^3 + n^3 && (\text{since } n \geq 10, \text{so } n^3 \geq 10n^2) \\
&\geq n^3 + 10n^2 = n^3 + 3n^2 + 7n^2 && (\text{since } n \geq 10, \text{so } 7n^2 \geq 70n) \\
&\geq n^3 + 3n^2 + 70n \\
&= n^3 + 3n^2 + 3n + 67n && (\text{since } n \geq 10, \text{so } 67n \geq 670) \\
&\geq n^3 + 3n^2 + 3n + 1 + 669 \\
&\geq n^3 + 3n^2 + 3n + 1 = (n+1)^3 && Q.E.D. \qquad \square
\end{aligned}
$$

2. Prove that $9^n - 8n - 1$ is divided by 64, for $n \in \mathbb{N}$.

The mistakes that students has done that besides want to jump to conclusions soon, students also do not understand in notions in mathematics, so they used it improperly, as reported by Moore (Maya, 2011).



L. Langkah Induksi:
Asumsikan benar untuk $P(n)$ sehingga $64 \mid 9^n - 8n - 1$ kar
maka terdapat bilangan bulat m sehingga $9^n - 8n - 1 =$
$9^n - 8n = 64m + 1$
Akan dibuktikan $P(n+1)$ maka
$9^{(n+1)} - 8(n+1) - 1 = 9^n \cdot 9 - 8n - 8 - 1$
$= 9^n - 8n$
$= 64m + 1$
Karena $9^{(n+1)} - 8(n+1) - 1$ bisa di nyatakan sebagai
maka terbukti bahwa $9^{(n+1)} - 8(n+1) - 1$ habis di

Alternative proof for induction step from examples 2 as follow:

$$
\begin{aligned}
9^{n+1} - 8(n+1) - 1 &= 9.9^{n} - 8n - 8 - 1 \\
&= 9(64k + 8n - 1) - 8n - 9 && (\text{with } k \in \mathbb{Z}) \\
&= 64(9k) + 72n - 8n + 9 - 9 \\
&= 64m + 64 && (\text{with } m = 9k) \\
&= 64(m+1) && (\text{can be divide } by\ 64) \quad \square
\end{aligned}
$$

Conclusion

- From the observation of students proving and their analysis, this study can be more developed.
- Even though teaching students to compose proofs is a difficult task, but is our challenge as instructor to make students do right proof.
- Taking lesson from students' proving mistakes, mathematics learning has to be planned as good as can be, so they can be used to enhance or to develop students ability in constructing proof or proving.

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THANK YOU